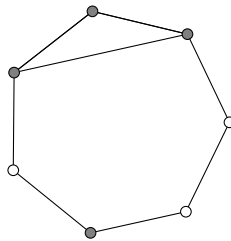


1. n people are at a party. Show that at least two people must have the same number of friends at the party (assume friendship is symmetric).
2. How many permutations of the 26 letters of the English alphabet contain none of the words fish, rat or bird?
3. (i) Fix some n and consider $A \subseteq \{1, 2, \dots, 2n\}$ with $|A| = n + 1$. Show that there exists at least two elements of A which are coprime.
(ii) (Harder!) Prove that there is at least one element of A which divides another.
4. Each of the vertices of a regular heptagon (seven sides) are coloured with two colours. Prove that there exists an isosceles triangle within it with all vertices the same colour.



5. How many natural numbers less than or equal to one million are neither a square, a cube nor a fifth power?
6. Let $N = \{1, 2, \dots, 100\}$ and $A \subset N$ with A of size 55.
 - (i) Show that there must be a pair of elements of A whose difference is 9.
 - (ii) (Bonus) Is the same result true if $|A| = 54$?