1. Find a nice form for the generating functions for the following sequences:
   (i) $(a_n) = (1, 2, 1, 0, 0, 0, \ldots)$,
   (ii) $(b_n) = (1, 2, 3, 4, 5, 6, \ldots)$,
   (iii) $(c_n) = (1, -2, 4, -8, 16, -32, \ldots)$,
   (iv) $d_n = \binom{2014}{n}$,
   (v) $e_n = (-1)^n n$.

2. Assume the generating function of a sequence $(a_0, a_1, a_2, \ldots)$ is $F(x)$. Find the generating functions of:
   (i) $(0, a_0, 0, a_1, 0, a_2, \ldots)$,
   (ii) $(a_0, 0, -2a_1, 0, 4a_2, 0, -8a_4, \ldots)$.

3. Find the sequences given by each of these generating functions:
   (i)
   $$\frac{1}{1-3x},$$
   (ii)
   $$\frac{2x-3}{1-7x+10x^2},$$
   (iii)
   $$\frac{x}{(1-x)^2}.$$

4. Solve the recurrence
   $$Q_n = \begin{cases} 0 & n \leq 0, \\ 1 & n = 1, \\ Q_{n-1} + 2 \cdot Q_{n-2} & n \geq 2, \end{cases}$$
   by first finding a generating function for Q_n.

5. Suppose you want to place 675 identical balls into four different bins, in such a way that each bin has either 150, 175 or 200 balls in it. Using a generating function, find the number of ways of doing this.

6. Suppose you have infinitely many red balls, one white ball and infinitely many blue balls.
   (i) Find a generating function for the number of distinct ways of picking n balls from this set.
   (ii) What is the generating function for this set if we make the additional restriction that an even number of blue balls must be taken? Simplify this function as much as you can and find the number of ways to pick the balls in this manner.